(This question paper contains 3 printed pages)

Sl. No. of QP: 8899

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(22)

Name of the Paper : MAHT-302—Numerical Methods and Programming

Name of the Course : B.Sc. (H) Mathematics

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Semester : III

Duration : 3 Hours

Maximum Marks : 75

(Write your Roll No. on the top immediately on receipt of this question paper.)

All the six questions are compulsory.

Attempt any two parts from each question.

Marks are indicated against each question.

Choice is given within the question.

Use of Scientific Calculator is allowed.

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1. (a) Perform three iterations of Newton's method to find a root of the equation $x^3 - 5x + 1 = 0$, considering the starting approximation as 0.5.

(b) Let f be a continuous function on the interval [a,b] and suppose that f(a)f(b) < 0. Prove that the bisection method generates a sequence of approximations $\{p_n\}$ which converges to a root p ∈ (a, b) with the property

$$|p_n - p| \leq \frac{b-a}{2^n}$$

(c) Verify that the function $f(x) = x^3 + 2x^2 - 3x - 1$ has a zero on the interval (1, 2). Perform four iterations of the bisection method.

(13)

2. (a) Verify that the equation $x^5 + 2x - 1 = 0$ has a root in the interval (0, 1). Perform three iterations of the secant method to approximate a root, considering $p_0 = 0$ and $p_1 = 1$.

(b) Perform three iterations of the false position method to approximate a root of the function $f(x) = cos(x) - x$ in the interval (0,1).

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(c) Define order of convergence of an iterative method. Find the order of convergence of Newton's method. **(13)**

3. (a) Find an LU decomposition of the matrix

$$A = \begin{bmatrix} 4 & 1 & 1 \\ 1 & 4 & -2 \\ 3 & 2 & -4 \end{bmatrix}$$

and use it to solve the system $AX = [4 \quad 4 \quad 6]^T$.

(b) Starting with the initial vector $X^{(0)} = (0,0,0)$, perform three iterations of the Gauss Seidal method to solve the system of equations, for the given coefficient matrix and the right hand side vector

$$\begin{bmatrix} 3 & 1 & 2 \\ -1 & 4 & 2 \\ 2 & 1 & 4 \end{bmatrix}, \qquad \begin{bmatrix} 6 \\ 5 \\ 7 \end{bmatrix}.$$

(c) Starting with the initial vector $X^{(0)} = (0,0,0)$, perform three iterations of the Jacobi method to solve the system of equations, for the given coefficient matrix and the right hand side vector

$$\begin{bmatrix} 5 & 1 & 2 \\ -3 & 9 & 4 \\ 1 & 2 & -7 \end{bmatrix}, \qquad \begin{bmatrix} 10 \\ -14 \\ 33 \end{bmatrix}.$$ **(13)**

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4. (a) Use Newton Divided difference Method to estimate $Sin(0.15)$ from the following data set

x	0.1	0.2
$f(x) = Sin(x)$	0.09983	0.19867

and also verify the theoretical error bound.

(b) Find the Lagrange interpolation polynomial that fits the data:
$f(-1) = -2,\ f(1) = 0,\ f(4) = 63,\ f(7) = 342.$

Hence interpolate at $x = 5.0$.

(c) Prove that for $n+1$ distinct nodal points $x_0, x_1, x_2, \cdots, x_n$ there exists a unique interpolating polynomial of at most degree n. **(12)**

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5. (a) Define the central difference operator (δ) and backward difference operator (∇).

Also prove that: $\nabla = -\frac{\delta^2}{2} + \delta\sqrt{1 + \frac{\delta^2}{4}}$.

(b) If $f(x) = e^{ax}$, then show by induction method that $\Delta^n e^{ax} = (e^{ah} - 1)^n e^{ax}$.

(c) Derive the following backward difference approximation formula for the first order derivative, where h is the spacing between the points.

$$f'(x_0) = \frac{1}{2h}(3f(x_0) - 4f(x_0 - h) + f(x_0 - 2h)) \qquad (12)$$

6. (a) Evaluate $\int_1^2 \frac{dx}{x}$ by Trapezoidal Rule and verify the theoretical error bound.

(b) Apply Euler's method to approximate the solution of the initial value problem

$\frac{dx}{dt} = \frac{t}{x}$, $0 \le t \le 3$, $x(0) = 1$, $h = 0.5$

(c) Verify that the forward difference approximation:

$$f'(x_0) = \frac{1}{2h}(-3f(x_0) + 4f(x_0 + h) - f(x_0 + 2h))$$

for the first order derivative provides the exact value of the derivative, regardless of the value of h, for the functions $f(x) = 1$, $f(x) = x$, $f(x) = x^2$ but not for the function $f(x) = x^3$. (12)

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